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Construction. Draw the diameter ON and extend it beyond N to P making $NP = 2l$. On OP as diameter construct the semi-circle $OSS'P$. On the given tangent lay off $OT = l$. Through T draw the line TSS' parallel to OP to meet the semi-circumference first in S. Drop a perpendicular from S to OP meeting OP in M and meeting the given circle in R and R'. Then R (or R') is the required point.

Proof. Let $QR = OM = x$, $OR = y$, $OS = z$, $ON = 2r$. Then by construction $OP = 2(r + l)$ and $MS = l$.

By elementary geometry $OM : OR = OR : ON$.

Hence

$$y^2 = 2rx. \tag{1}$$

Similarly $$OM : OS = OS : OP.$$

Hence, $$z^2 = 2x(r + l), \tag{2}$$

or, from (1), $$z^2 = y^2 + 2lx. \tag{3}$$

But $z^2 = x^2 + l^2$, and therefore (3) becomes

$$y = \pm (l - x). \tag{4}$$

Since $y > 0$ and $l > x$, the lower sign is inadmissible. Hence $x + y = l$, or

$$QR + OR = l.$$

A solution exists when and only when $l \leqq 4r$, and there is not more than one solution on the same side of the diameter through the point of contact.

Solutions were also received from WALTER C. EELLS, H. C. FEEMSTER, C. N. SCHMALL, ELMER SCHUYLER, and A. H. HOLMES.

420. Proposed by C. N. SCHMALL, New York City.

Four spheres are described so that each touches a face of a given triangular pyramid and the other three faces produced. If the radii of the escribed spheres be r_1, r_2, r_3, r_4, and r be the radius of the inscribed sphere, show that

$$\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4} = \frac{2}{r}.$$

SOLUTION BY HORACE OLSON, Chicago, Illinois.

Let $ABCD$ be a given tetrahedron.

Let the faces ABC, ABD, ACD, and BCD be represented by small letters corresponding to the opposite vertices of the tetrahedron, *i. e.*, by d, c, b, and a respectively.

Let V represent the volume of the tetrahedron $ABCD$.

Let O be the center, and r the radius, of the inscribed sphere.

Let O_1, O_2, O_3, and O_4 be the centers, and r_1, r_2, r_3, and r_4 the radii, of the escribed spheres opposite to A, B, C, and D, respectively.

Connect points O and O_3 with the 4 vertices of the tetrahedron.

Then,

$$V = AOBC + OBCD + OACD + OABD = \frac{ar}{3} + \frac{br}{3} + \frac{cr}{3} + \frac{dr}{3} = \frac{r}{3}(a + b + c + d),$$

and

$$r = \frac{3V}{a+b+c+d}.$$

Also

$$V = AO_3BC + O_3BCD + O_3ACD - O_3ABD = \frac{r_3}{3}(a+b-c+d)$$

and

$$r_3 = \frac{3V}{a+b-c+d}.$$

Similarly,

$$r_1 = \frac{3V}{(-a+b+c+d)}, \quad r_2 = \frac{3V}{(a-b+c+d)}, \quad \text{and} \quad r_4 = \frac{3V}{(a+b+c-d)}.$$

Therefore

$$\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}+\frac{1}{r_4} = \frac{(-a+b+c+d)+(a-b+c+d)+(a+b-c+d)+(a+b+c-d)}{3V}$$

$$= \frac{2a+2b+2c+2d}{3V} = \frac{2}{r}.$$

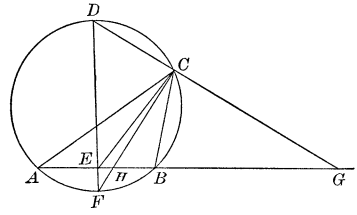
Also solved by A. M. HARDING and ELMER SCHUYLER.

422. Proposed by J. SCHEFFER, Hagerstown, Md.

Construct strictly by use of elementary plane geometry, a triangle having given the product of two sides, the median to the third side, and the difference of the angles to the third side.

SOLUTION BY F. M. MORGAN, Dartmouth College.

Suppose ABC is the required triangle circumscribed by a circle. Draw the internal and external bisectors of $\angle C$ meeting the circle in points F and D. Draw FD.



Since $\angle DCF$ is a right angle, FD is a diameter and bisects AB. Produce DC to meet AB in G. Since $\angle FEG = \angle FCG = 90°$, a circle will pass through points F, E, C, G. Therefore, $\angle EFH = \angle EGC = \frac{1}{2}$ of the difference of the base angles of triangle ABC. This angle is given and will be called α.

Now $\angle EHC = 90° + \alpha$ and is therefore known. Moreover, $CH \cdot CF = CA \cdot CB$ which is a given product.

Construction. Take line CE equal to the given median. Upon it describe the segments of two circles, one to contain an angle α, and the other an angle $90°+\alpha$. Draw line through C cutting these circles so that $CH \cdot CF = CA \cdot CB$. To do